SOLUTIONS OF QUESTIONS

IN

A COURSE OF

NATURAL PHILOSOPHY.

BY

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MDCCCLXXIII.

SOLUTIONS OF EXERCISES.

STATICS.

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(1.) 18 lbs. : 168 lbs. : : 3 ft. : 28 feet.

(2.) 36 in. : 17 in. : : 18 lbs. : $8\frac{1}{2}$ lbs.

(3.) The larger force is $\frac{4}{5}$ of the single force, therefore the smaller force is $\frac{1}{5}$ of the single force.

(4.) 64 in. : 7 in. : : 20 cwt. : 2 cwt. 21 lbs.

(5.) 6 cwt. : $\frac{720}{1728}$ cwt. of 6 : : 12 in. : 5 inches.

(6.) It is required to find two numbers the sum and difference of which are respectively 12 and 2.

$x + y = 12$; $x - y = 2$ $\therefore$ $x = 7$, $y = 5$.

(7.) Tensions are 4, $4 + 5 = 9$, $4 + 5 + 7 = 16$. 9 lbs. : 4 lbs. : : $11\frac{1}{4}$ in. : 5 inches; 9 lbs. : 16 lbs. : : $11\frac{1}{4}$ in. : 20 inches.

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(12.) $R = \sqrt{63^2 + 16^2} = 65$.

(13.) $R = \sqrt{84^2 + 13^2} = 85$.

(14.) $Q = \sqrt{(113 + 112)\ (113 - 112)} = \sqrt{225 \times 1} = 15$.

(15.) Here $\sqrt{(33a)^2 + (56a)^2} = 65a.$
But $65a = 130 \therefore a = 2$
wherefore $P = 33 \times 2 = 66$, and $Q = 56 \times 2 = 112$.

(16.) $P = \sqrt{(85 + 77)(85 - 77)} = \sqrt{162 \times 8} = 36.$

(17.) $Q = \sqrt{(30.5 + 27.3)(30.5 - 27.3)} = \sqrt{57.8 \times 3.2} = 13.6.$

(18.) Let x = resultant; larger force $= 259.2 - x$.
$x^2 = 7.2^2 + (259.2 - x)^2.$
$x^2 = 51.84 + 51.84 \times 1296 - 518.4\,x + x^2.$
$\therefore x = 129.7;\ 259.2 - 129.7 = 129.5.$

(19.) Complete the parallelogram. It will be seen that the oblique force is inclined at an angle of 45°, and is equal to the square root of twice the vertical force.

(20.) Let $P = 16a$, then $Q = 63a$; hence $R = \sqrt{(16a)^2 + (63a)^2} = 65a$. But $65a = 13$, therefore $a = \frac{1}{5}$. Therefore $\frac{16}{5} = 3.2$ and $\frac{63}{5} = 12.6$ are respectively the two forces.

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(1.) From formula Sec. 24. $R^2 = P^2 + P^2 + P^2 = 3\,P^2 = 3 \times 15^2 \therefore R = 15\sqrt{3}.$

(2.) $R^2 = P^2 + P^2 + P^2 \times \sqrt{3} = 20^2 + 20^2 + 20^2\sqrt{3} = 692.8 \therefore R = 26.32.$

(3.) $R^2 = P^2 + P^2 + P^2 \times \sqrt{2} = 40^2 + 40^2 + 40^2\sqrt{2} = 5462.4 \therefore R = 73.9.$

(4.) See Section 20, II. deduction.

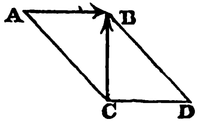
(5.) $R^2 = P^2 + P^2 - P^2\sqrt{2} = 100^2 + 100^2 - 100^2\sqrt{2} = 5860 \therefore R = 76.5.$

(6.) $R^2 = P^2 + Q^2 - PQ = 10^2 + 42^2 - 420 = 1444 \therefore R = 38.$

(7.) $R^2 = P^2 + Q^2 + PQ = 8^2 + 12^2 + 96 = 304 \therefore R = 17.43.$

(8.) $R^2 = P^2 + Q^2 - PQ = 9^2 + 11^2 - 99 = 103 \therefore R = 10.14.$

(9.) Let A B and C B represent P and R respectively. Complete the parallelogram. D B represents force Q, and being the hypothenuse of $\triangle$ B D C, is greater than B C.



(10.) $R^2 = P^2 + P^2 - P^2\sqrt{3} = 2 \times 50^2 - 50^2 \times \sqrt{3} = 670 \therefore R = 25.88.$

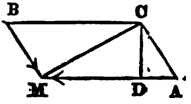
(11.) At 120°. See Section 20, II. deduction.

(12.) $R^2 = P^2 + P^2 + P^2 = 3P^2$
$3P^2 = 8100$; $P^2 = 2700 = 900 \times 3 \therefore P = 30\sqrt{3}.$

(13.) Construct the figure (see fig. in Ex. 9 above). Horizontal force equals the vertical force. Oblique force equals square root of twice the vertical force.

(14.) $R^2 = P^2 + P^2 = 2P^2$ and $P^2 = \frac{10^2}{2} = 50 \therefore P = 7.07.$

(15.) $R^2 = P^2 + P^2 - P^2\sqrt{2} = P^2(2 - \sqrt{2}) = P^2 = \frac{100}{2 - \sqrt{2}} = \frac{100(2 + \sqrt{2})}{2} = 170.7 \therefore P = 13.065.$



(16.) See Figure. $CD = \frac{55}{2}$, $DA = 27.5\sqrt{3}.$

$MD = \sqrt{(95 + 27.5) \times (95 - 27.5)} = \sqrt{122.5 \times 67.5}$
$= \sqrt{49 \times 5 \times .5 \times 9 \times 3 \times 5 \times .5} = \sqrt{7^2 \times 5^2 \times .5^2 \times 3^2 \times 3}$
$= 7 \times 5 \times .5 \times 3\sqrt{3} = 52.5\sqrt{3}.$

Then $(MD + DA)$ the other component $= 52.5\sqrt{3} + 27.5\sqrt{3} = 80\sqrt{3}.$

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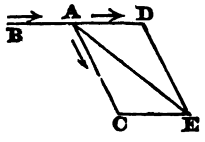
(1.) See Section 30.

(2.) Two of the forces are in the same straight line, and neutralize each other, leaving as resultant the third force.

(3.) Here the two equal forces P and R act at an angle of 120°. Their resultant is, therefore, equal to one of them, and bisecting the angle acts in the same direction as the force Q. Total resultant is therefore 10 lbs.

(4.) If we take the forces in order, we find that the first and third forces are of 5 lbs. each, are exactly opposite to other forces of 5 lbs., and are therefore neutralized. We have, also, three forces of 8 lbs. each, acting at right angles to each other. Two of them act in opposite directions in the same straight line, and are therefore neutralized. We have then left as resultant one force of 8 lbs.



(5.) Let B A and A C represent the two forces. The pushing force, or thrust, B A, may be replaced by the equal drawing force, or strain, A D. Complete the parallelogram. The diagonal A E represents the resultant.

(6.) $\sqrt{119^2 + 120^2} = \sqrt{28561} = 169.$

Since resultant 169 is equal to square root of the sum of the two forces, therefore the angle between them must be a right angle.

(7.) $$R^2 = P^2 + P^2 = 2\,P^2$$

$$P^2 = \frac{100}{2} = 50 = 25 \times 2$$

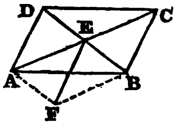
$$P = 5\,\sqrt{2}.$$

(8.) $$\sqrt{44^2 + 117^2} = \sqrt{15625} = 125$$

$$\therefore\ AB = 125.$$

Now, if a triangle A′ B′ C′ be drawn equal in all respects to A B C turned through 90°, its sides will be parallel to, and therefore proportional to, the forces. Hence, 125 in. represents a weight of 10 lbs., therefore 1 in. represents ·08 lbs.; 44 inches represent $44 \times {\cdot}08 = 3{\cdot}52$ lbs.; and 117 represents $117 \times {\cdot}08 = 9{\cdot}36$ lbs.

(9.) Let A C, B D be the diagonals intersecting in E. On A E, B E make the parallelogram A E B F, and join E F. Because D E is parallel and equal to A F, therefore F E is parallel and equal to A D. Now, forces represented by A E, B E will be half the forces represented by A C, B D; and F E, the resultant of the former, will therefore be half the resultant of the latter. Hence, this resultant is twice the force represented by A D.



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(1.) See Section 38.

(2.) The magnitude of resultant = (18 + 12) = 20 lbs.

Let x be the distance of its point of application from a.

$$\therefore 20x = 12 \times 12$$
$$\text{and } x = 7{\cdot}2.$$

(3.) See Section 41.

(4.) See Section 44.

(5.) $x \times 30 = 5 \times 24$

$$\therefore x = 4.$$

(6.) $\frac{10}{22}$ of 66 = 30 lbs., and 66 — 30 = 36 lbs. = the larger force.

$$36 \text{ lbs.} : 30 \text{ lbs.} :: 42 \text{ in.} : x$$
$$\therefore x = 35 \text{ in.} = 2 \text{ ft. } 11 \text{ in.}$$

(7.) Resultant = (112 × 20).

Let x = distance of smaller force from resultant.

$$\therefore 132 \times x = 112 \times 32$$
$$\text{or } x = 28.$$

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(8.) 112 — 35 = 77 = the other force.

Let x = the distance of the smaller force from the resultant,

$$\therefore 112 \text{ lbs.} : 77 \text{ lbs.} :: 64 \text{ in.} : x \text{ inches}$$
$$\therefore x = 44 \text{ inches}$$

and 64 — 44 = 20 inches = Distance of larger force from resultant.

(9.) 154 — 99 = 55 lbs. = P. Then $\dfrac{63 \times 55}{99}$ = 35 = C B. 63 in. + 35 in. = 98 inches = A B.

(10.) 8 : 3 :: 16 in. : x in. ∴ B C = 6; and A C = 16 — 6 = 10 in. 16 in. : 10 in. :: 104 lbs. : 65 lbs. = Q; and 104 — 65 = 39 lbs. = P.

(11.) Let x = distance in inches of the point

from the end where acts the 1 lb. The weights are 5 inches apart, therefore
$(1+2+3+4+5)x = 1 \times 0 + 2 \times 5 + 3 \times 10 + 4 \times 15 + 5 \times 20$;

$$15x = 200, \text{ and } x = 13\tfrac{1}{3} \text{ inches.}$$

(12.) One man is 2 feet, the other 3 feet, from the middle. Hence, if P and Q be the required weights,

$$P + Q = 1{\cdot}5, \text{ and } 2P = 3Q$$
$$\therefore P = {\cdot}9; \; Q = {\cdot}6 \text{ cwt.}$$

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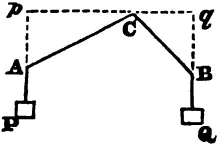
(1.) If the forces were in equilibrium, the sum of the moments about any point would be zero. But this is not the case; for, consider the point A. The moments of forces along A B and C A about A are zero because they pass through A; but the moment of the remaining force along B C is not zero.

(2.) See Sections 48—52.

(3.) The moments of two of the forces about the fixed point are zero by hypothesis; hence, in order that there may be equilibrium, the moment of the third must be zero. Hence, its direction must also pass through the fixed point.

(4.) Moment of $P = P \times pC = P \times \frac{AC}{2}\sqrt{3}$

$$\text{,,} \quad \text{,,} \quad Q = Q \times qC = Q \times \frac{BC}{2}\sqrt{2}$$



Since the angles p C A and q C B are respectively 30° and 45°,

$$\therefore P \times \frac{AC}{2} \sqrt{3} = Q \times \frac{BC}{2} \sqrt{2}, \text{and } P \sqrt{3} = Q \sqrt{2}$$

$$\therefore 3 P^2 = 2 Q^2, \text{ whence } P^2 : Q^2 :: 2 : 3.$$

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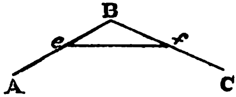
(1.) Let x = distance of the C. G. from the lighter weight.

Then $11 x = 8 \times 33$, or $x = 24$ inches.

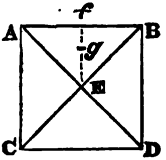


(2.) Let A B C be the bent wire. Bisect A B in e, and B C in f. Then we may suppose the weight of A B to be 2, and to act at e, and the weight of B C to act at f. Hence, the Centre of Gravity is $\frac{1}{3}$ of $e f$ from e.

(3.) 60 lbs. : 72 lbs. : : 5 in. : x in. Distance required $= 6 + 5 = 11$ inches.

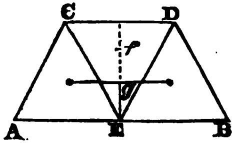
(4.) Let x = distance in inches from larger weight. Then $(2 + 4 + 6 + 8) x = 8 \times 0 + 6 \times 6 + 4 \times 12 + 2 \times 18 = 120.$

$$20 x = 120, \text{ and } x = 6 \text{ inches.}$$



(5.) Let triangle C E D be taken away; then the C. G. of two triangles, A E C and E B D, is at the centre E. The C. G. of triangle A B E is at g, $\frac{2}{3}$ of $\frac{a}{2} = \frac{a}{3}$ from the centre. We have, therefore, 2 triangles at E, and 1 at g. Let x = distance of the C. G. required from E $\therefore 1 \times \frac{a}{3} = 3 \times x$, and $x = \frac{a}{9}$.

(7.) The C. G. of the triangles A C E, E D B, bisects the line joining their C. G.'s, which is $\frac{1}{3}$ of h the perpendicular height of the figure from the base, the C. G. of triangle C E D is at f, $\frac{2}{3}$ h from base. Then $g f = \frac{1}{3} h$. Now, we have 2 triangles at g, and 1 at f. Hence, taking moments about E,



$$2 \times \tfrac{1}{3} h + 1 \times \tfrac{2}{3} h = 3 \times x \;\therefore\; x = \tfrac{4}{9} h.$$

(8.) See Section 58. Let w = weight of board. Since three weights, each $\frac{w}{3}$ at the corners, will have a resultant w at the C. G., the board will be supported if each man bears one-third of the weight.

(9.) Find the C. G. of the triangle. Bisect the line joining this point and the angle to which the weight is suspended.

(10.) The C. G. of the triangle is distant from the base $\frac{1}{3}$ of the length of the line joining the middle of the base with the apex. The C. G. of the two weights is at the middle of the base. Then, by taking moments about the middle point of the base, it will be found that the C. G. is $\frac{1}{6}$ of the middle line measured from the base.

(11.) Take moments about the two adjacent sides of the square, and let x and y be the distances of the C. G. from these sides. Then

$$2 \times 4 + 5 \times 4 + 3 \times 0 + 4 \times 0 = 14 \times x \;\therefore\; x = 2,$$
$$\&\; 2 \times 4 + 3 \times 4 + 4 \times 0 + 5 \times 0 = 14 \times y \;\therefore\; y = \tfrac{10}{7}$$

Hence the distance of the C. G. from the centre $= 2 - \frac{10}{7} = \frac{4}{7}$.

(12.) $4 + 3 + 2 \cdot 4 = 9 \cdot 4 =$ height of the triangle.

$$\begin{aligned} &\text{ft.} \quad \text{ft.} \quad \text{lbs.} \\ &9{\cdot}4 : 4 :: 94 : x \therefore x = 40 \text{ lbs.} \\ &9{\cdot}4 : 3 :: 94 : y \therefore y = 30 \text{ lbs.} \\ &9{\cdot}4 : 2{\cdot}4 :: 94 : z \therefore z = 24 \text{ lbs.} \end{aligned}$$

(13.) Let x, y, z be the distances of the C. G. from the three sides, and h their sum, which is equal to the height of the triangle. Take moments about each side in succession.

$$1 \times h + 2 \times 0 + 3 \times 0 = 6x \therefore x = \tfrac{1}{6} h$$
$$2 \times h + 1 \times 0 + 3 \times 0 = 6y \therefore y = \tfrac{1}{3} h$$
$$3 \times h + 1 \times 0 + 2 \times 0 = 6z \therefore z = \tfrac{1}{2} h$$

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(1.) (30 lbs. + 20 lbs.) : 30 lbs. :: 15 ft. : x
$\therefore x = 9$ ft., and $15 - 9 = 6$ ft.

(2.) Then $7 \times 3 = 9 \times x \therefore x = 2\frac{1}{3}$ lbs.

(3.) Let $l =$ the length of the beam. Then
$\frac{1}{5} l \times \text{W} = l \times \text{P} \therefore \text{P} = \frac{1}{5}\text{W} = 10$ lbs.
Pressure on beam $= \text{W} - \text{P} = 50 - 10 = 40$ lbs.

(4.) The pressure on one prop due to the additional weight $= 10 - 4 = 6$ lbs.
Then 36 lbs. : 6 lbs. :: 54 in. : x inches from the prop $\therefore x = 9$.

(5.) Let $x =$ the distance of the fulcrum from the larger weight; then $9 \times 5 = 15x \therefore x = 3$ ft., and $9 + x = 12$ ft. Now, the increased weights are 20 and 10; hence, if $y =$ the distance of the new fulcrum from the larger weight, $30 \times y = 10 \times 12 \therefore y = 4$ feet.

(6.) Here $100 \times 18 = \text{P} \times (42 + 18) \therefore \text{P} =$ 30 lbs.

(7.) Let x = feet from end to which weight is suspended; let w = the weight of the beam.

$$2\,w\,x = (\frac{12}{2} - x) \times w;\ 3\,x = 6 \text{ and } x = 2.$$

(8.) $W \times 1{\cdot}4 = 56\ (10 - 3 - 1{\cdot}4) = 313{\cdot}6$ and $W = 224$ lbs.

(9.) Two forces whose difference is 7 lbs. and sum 13 lbs. will be easily found to be 3 lbs. and 10 lbs. Let x = distance of the greater force from the fulcrum. Then $10\,x = 3\,(x + 14)\ \therefore x = 6$ in. and $6 + 14 = 20$ inches.

(10.) See Section 77.

$\sqrt{140 \times 154{\cdot}35} = \sqrt{4 \times 35 \times 35 \times 4{\cdot}41} = 2 \times 35 \times 2{\cdot}1 = 147$ grs.

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(11.) Let x = weight. Then $\cdot 98\,x = 14{\cdot}7$ Hence $x = 15$ kilos.

(12.) Weight of the body $= 7{\cdot}9 - 3 - 2 = 2{\cdot}9$. The ratio of the arms is inversely as the weights of the bodies; that is, as $3 : 2{\cdot}9 = 30 : 29$.

(13.) Let x = the distance in inches of the movable weight from the fulcrum.

First, to mark the position of the movable weight for 1 lb. $x \times 1 = 1 \times 8 \pm 1 \times 3\ \therefore x = 11$ inches or 5 inches, according as the C. G. is on the same or the opposite side of the fulcrum as the body to be weighed.

For 2 lbs. $x \times 1 = 2 \times 8 \pm 1 \times 3\ \therefore x = 19$ or 13.

For every additional 1 lb. we require the movable weight to be removed 8 inches farther from the fulcrum; we have then

11, 19, 27, etc., to 99;
or 5, 13, 21, etc., to 93.

(14.) Let W = weight of body, and p the distance of its point of suspension from the fulcrum. Let s = weight of steelyard, and c the distance of its C. G. from the fulcrum. Let w = the movable weight, and x its distance.

$$W \cdot p \pm s \cdot c = w \cdot x$$

$$\therefore x = \frac{W \cdot p \pm s \,.\, c}{w}$$

Hence, the quantities in the enumerator remaining the same, x varies inversely as w.

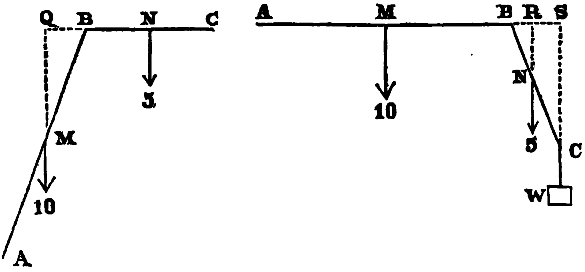
(15.) Since $P{\cdot}R = Q{\cdot}r \therefore 24 \times 8 = 12 \times r$.
Radius of the axle = 16 in.

(16.) $P \times 36 = Q{\cdot}r = 30 \times 15 \therefore P = 12{\cdot}5$ cwts.

(17.) $P \times 12 = Q{\cdot}r = 60 \times 5 \therefore P = 25$ lbs.

(18.) $P = 7\frac{1}{2} \times 2 = 15$ cwts. Let Q = the tension. Then $15 \times 144 = Q{\cdot}r = Q \times 18 \therefore Q =$ 120 cwts. = 6 tons.

(19.) Let A B be the larger arm, and B C the shorter. As the lever is uniform, we may consider the lengths of these arms to be 10 and 5 units respectively. Let M be the C. G. of A B, and N of B C.



In the first position, let B C be horizontal, and let the verticals through M meet C B produced in Q. Let Q B = x. By taking moments about B we have $10\ x = 5 \times \frac{5}{2}$ and $\therefore\ x = 1{\cdot}25$.

In the second position, let a weight W be at C, and let A B be horizontal. Let the verticals through C and N meet A B produced in S and R. Then $\triangle$ B S C in the second figure is equal to $\triangle$ B Q M in the first. $\therefore$ B S = x, and B R = $\frac{1}{2}\,x$.

Take moments about B.

$$\therefore W \times BS + 5 \times BR = 10 \times BM$$

$$\therefore W \times 1{\cdot}25 + 5 \times \frac{1{\cdot}25}{2} = 10 \times 5,$$

$$\text{and } \therefore W = 37{\cdot}5.$$

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(4.) See Section 87. Since there are 3 sheaves, there are 6 strings each bearing $\frac{1}{6}$ of weight. P therefore equals $\frac{1}{6}$ of a ton.

(5.) Since there are 4 sheaves, and 8 strings, 20 lbs. will support 160 lbs. Hence, the block weighs 160 — 140 = 20 lbs.

(6) See Section 86.

$$\begin{array}{r}
395 \\
7 \\
\hline
2)402 \\
\hline
201 \\
3 \\
\hline
2)204 \\
\hline
102
\end{array}
\qquad\qquad
\begin{array}{r}
102 \\
8 \\
\hline
2)110 \\
55 \\
\hline
5 \\
\hline
2)\ 60 \\
\hline
30
\end{array} = P.$$

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(7.) Pressure on the beam $= 395 + 30 + (7 + 3 + 8 + 5) = 448$ lbs.

(8.) To have the heaviest pulleys in the lowest position. For, let W, W′, W″, etc., be the weights of the pulleys in order from the highest, then it will be seen that P has to support $\frac{1}{2}$ W, $\frac{1}{4}$ W′, $\frac{1}{8}$ W″, and so on. Hence, the lower the pulley, the smaller the fraction of its weight, which is borne by P. The most advantageous arrangement is, therefore, that in which the heaviest is lowest, and all are arranged in order of weight.

(9.)

$$\begin{array}{r} 333 \\ 3 \\ \hline 2)336 \\ \hline 168 \\ 2 \\ \hline 2)170 \\ \hline 85 \\ 1 \\ \hline 2)\ 86 \\ \hline 43 \end{array} = \text{P.}$$

Then $154 - 43 = 111 =$ pressure exerted on the floor by the man.

(10.) 154 — 100 = 54 = Power exerted by the man.

$$\begin{array}{rl} 54 & \times\ 2 \\ \hline 108 & \\ 1 & \\ \hline 107 & \times\ 2 \\ \hline 214 & \\ 2 & \\ \hline 212 & \times\ 2 \\ \hline 424 & \\ 3 & \\ \hline 421 & \text{Weight sustained.} \end{array}$$

Beginning with highest

(11.) $R^2 = P^2 + P^2 + P^2 = 3\ P^2$

$$\therefore P^2 = \frac{8100}{3} = 2700$$

$$\text{And } P = 30\ \sqrt{3}.$$

(12 and 13.) See Section 87, Fig. 81.

(14.) The tensions of the strings, which together support the weight, are

$$\left.\begin{array}{l} P \\ 2\ P + 1\frac{1}{2} \\ 4\ P + (1\frac{1}{2} \times 2) + 2\frac{1}{2} \end{array}\right\} = 147$$

$$\therefore 7\ P = 140, \text{ and } P = 20 \text{ lbs.}$$

(15.) The various tensions are

$$\begin{array}{rll} & P & \\ 2 & P & + 2 \\ 4 & P & + 2 \times 2 + 3 \\ 8 & P & + 4 \times 2 + 2 \times 3 + 4 \\ 16 & P & + 8 \times 2 + 4 \times 3 + 2 \times 4 + 5 \\ \hline \end{array}$$

$$\therefore W = 31 \times 12 + 15 \times 2 + 7 \times 3 + 3 \times 4 + 5 = 440 \text{ lbs.}$$

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(1.) 1. $P = \dfrac{100 \times 3}{4} = 75$ lbs.

2. Length of plane $= \sqrt{11^2 + 60^2} = \sqrt{3721}$ $= 61$.

Then 61 in. : 11 in. : : 122 lbs : P $\therefore$ P = 22 lbs.

$R = \sqrt{(W^2 - P^2)} = \sqrt{(122 + 22)(122 - 22)}$

$= \sqrt{144 \times 100} = 120$ lbs.

3. $W = \dfrac{35 \times 27}{15} = 63$ lbs.

4. Here $\dfrac{P}{R} = \dfrac{\text{height}}{\text{base}}$

Since P is equal to R in this case, the base is equal to the height, and therefore the angle B A C, that is, the inclination of the plane, is 45°.

5. Length $= \sqrt{32^2 + 255^2} = \sqrt{66049} = 257$

$W = \dfrac{160 \times 257}{32} = 5 \times 257 = 1285$ lbs.

6. Length $= \sqrt{72^2 + 1295^2} = \sqrt{1682209}$ $= 1297$.

$P = \dfrac{14267 \times 72}{1297} = 72 \times 11 = 792$ lbs.

7. $P = \dfrac{50 \times 1}{50} = 1$ ton.

(2.) 1. $P = \dfrac{15 \times 6}{9} = 10$ lbs.

2. The base $= \sqrt{18 \times 8} = \sqrt{9 \times 2 \times 4 \times 2} = 12$

$P = \dfrac{36 \times 5}{12} = 15$ lbs.

3. Height $= \sqrt{49 \times 1} = 7$

$W = \dfrac{3.5 \times 24}{7} = 12$ lbs.

4. $P = \frac{9 \times 28}{45} = 5 \cdot 6$

$R = \sqrt{5 \cdot 6^2 + 9^2} = \sqrt{112 \cdot 36} = 10 \cdot 6.$

5. See Fig. 82. Since $bc = \frac{1}{2} ab$, therefore by § 22 the angle bac is 30°; wherefore also the angle B A C, the inclination of the plane, is 30°.

6. When P acts parallel to the plane, and supports W,

$$\frac{P}{W} = \frac{h}{l} \text{ or } W = \frac{l}{h} \cdot P$$

When 2 P, acting horizontally, supports W

$$\frac{2P}{W} = \frac{h}{b} \text{ or } W = \frac{2b}{h} \cdot P$$

$$\therefore \frac{l}{h} \cdot P = \frac{2b}{h} \cdot P$$

$\therefore l = 2b$, and by § 22, B A C = 60°.

(3.) $\sqrt{225 \times 49} = 15 \times 7 = 105 =$ Base.

A horizontal force P will support a weight equal to

$$P \times \frac{\text{base}}{\text{height}} = \frac{P \times 105}{88}$$

A force P parallel to the plane will support a weight equal to

$$P \times \frac{\text{length}}{\text{height}} = \frac{P \times 137}{88}$$

But the whole weight supported is 253 lbs.

$$\therefore P \times \frac{105 + 137}{88} = 253$$

$$P = \frac{253 \times 88}{242} = 92 \text{ lbs.}$$

(4.) The base $= \sqrt{242 \times 200} = \sqrt{11^2 \times 2 \times 10^2 \times 2}$ $= 220$

$$\text{Then } W = \frac{84 \times 220}{21} = 880$$

$$\text{Hence total weight } = 880 + 20 = 900.$$

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(5.) Since, by reference to Fig. 83, we see $b\ c$ is $\frac{a\ b}{2}$ when angle a is 30°

Tension : Weight as 1 : 2.

$\therefore$ Tension = 40 lbs.

(6.) 100 lbs. See Fig. 82; $b\,c = a\,c$.

(7.) See Section 91. $\frac{P}{W} = \frac{d}{c}$

$$\therefore P = 110 \times \frac{\frac{1}{4}}{7 \times \frac{22}{7}} = \frac{110}{88} = 1{\cdot}25 \text{ lbs.}$$

(8.) $W = P \times \frac{c}{d} = 10 \times \frac{12}{\frac{1}{3}} = 360$ lbs.

(9.) $\frac{P}{W} = \frac{d}{2xl}$. See Section 91.

$$P = \frac{352 \times \frac{3}{5}}{2 \times \frac{22}{7} \times 48} = \frac{352 \times 3 \times 7}{2 \times 22 \times 48 \times 5} = {\cdot}7 \text{ lbs.}$$

(10.) $W = 10 \times \frac{120}{\frac{2}{3}} = 1800$ lbs.

(11.) Let x = the distance between the threads.

$$\therefore \frac{P}{8P} = \frac{x}{4} \text{ and } x = \tfrac{1}{2} \text{ inch.}$$

Hence in 10 in. we have 20 turns.

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(1.) See Section 94.

$$P{\cdot}C = Q{\cdot}c$$

$$\therefore P \times 96 = 130 \times 6$$

$$\therefore P = \frac{130 \times 6}{96} = 8 \text{ lbs. } 2 \text{ ozs.}$$

(2.) Draw a system of six movable pulleys, as in Fig. 78.

$$P \cdot 2^6 = W. \quad \text{Then } W = 10 \cdot 2^6 = 640 \text{ lbs.}$$

(3.) Work done by B $= 4 \times 2 = 8$

Work done by A $= 3 \times (2 + 2 + 2) = 18$

Total work $= 18 + 8 = 26$ lbs.

(4.) Let W ascend 1 ft.; then B descends 1 ft., A descends 3 ft., and P descends 7 ft.

$$\therefore W = 20 \times 7 + 3 \times 3 + 4 \times 1$$

$$\therefore W = 153.$$

(5.) Length of plane $= \sqrt{5{\cdot}2^2 + 67{\cdot}5^2} = 67{\cdot}7$

Then $W \times 5{\cdot}2 = 13 \times 67{\cdot}7$

$$\therefore W = \frac{13 \times 67{\cdot}7}{5{\cdot}2} = 169{\cdot}25.$$

(6.) Work done by P $= 20 \times 3$

Work done by W $= W \times 2$

$W \times 2 = 20 \times 3$; whence $W = 30$ lbs.

(7.) Work done by P $= 14 \times 80$

Work done by W $= 2240 \times x$

$$2240 \times x = 14 \times 80$$

$$x = \frac{14 \times 80}{2240} = \tfrac{1}{2} \text{ inch.}$$

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(8.) There are 8 strings, each of which is shortened 6 in. $\therefore 6 \times 8 = 48$ in. $= 4$ ft. will pass through the hands.

(9.) There are 6 strings to the block, and one to the man's hand. Hence the total length of cord required will be $6 \times 30 + 30 = 210$ feet.

(10.) Suppose the weight to ascend 1 foot, then one foot of cord passes over the first of the movable

pulleys. Since two parts of the cord are shortened one foot, two feet of cord pass over the first fixed pulley.

Hence, if the second movable pulley remained stationary, 2 ft. would pass over it; but, since it rises one foot, 3 ft. pass over it. Since, up to the second fixed pulley, four parts of the string are shortened one foot, 4 ft. pass over this pulley. Therefore 5 ft. pass over the next movable pulley. Consequently, in order that the pulleys may all revolve in the same time, the circumferences of the fixed pulleys must be as 2 : 4 : 6, and the circumferences of the movable pulleys as 1 : 3 : 5.

(11.) Work done by $P = P \times 108$

$$\text{Work done by } W = 144 \times \tfrac{1}{2}$$

$$P \times 108 = 144 \times \tfrac{1}{2}$$

$$P = \frac{144 \times \frac{1}{2}}{108} = \tfrac{2}{3} \text{ lbs.}$$

(12.) $\sqrt{(85 + 13)(85 - 13)} = \sqrt{98 \times 72} = \sqrt{49 \times 2 \times 36 \times 2} = 84 =$ base.

Suppose the weight to be moved from the bottom to the top of the plane; then

$$\text{Work done by } P = 52 \times 84$$

$$\text{Work done by } W = W \times 13$$

$$13\,W = 52 \times 84; \text{ and } W = 336 \text{ lbs.}$$

DYNAMICS.

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(1.) Acceleration $= 90 \div 4\frac{1}{2} = 20$.

(2.) See Section 8. $79 - 30 = 49$ velocity gained in 7 seconds. Then $49 \div 7 =$ acceleration.

(3.) See Sections 8, 9, 10, 14.

(4.) Mean velocity $= \frac{1}{2}(43 + 17) = 30$

Space passed over $= 30 \times 5 = 150$.

See Figs. 99 and 101.

(5.) See Sections 13, 14.

(6.) Velocity at the end of 7 seconds $= 80 - (10 \times 7) = 10$.

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(7.) By formula $s = Vt - \frac{1}{2}ft^2$
$= 80 \times 7 - (\frac{1}{2} \times 10 \times 49) = 315$ feet.

(8.) $s = \frac{1}{2}ft^2 = \frac{1}{2} \times 32 \times 64 = 1024$ feet.

(9.) $s = Vt + \frac{1}{2}ft^2 = (11 \times 8) + (\frac{1}{2} \times 8 \times 64$
$= 344$ feet.

$v = (8 \times 8) + V = 64 + 11 = 75$ feet.

(10.) Let $x =$ the initial velocity.

Mean velocity $= 105 \div 5 = 21$ feet.

Then $(35 + x) \div 2 = 21$.

$\therefore x = 7$.

$\therefore$ Velocity gained in 5 seconds $= 35 - 7 = 28$ ft.

Hence acceleration $= 28 \div 5 = 5{\cdot}6$.

(11.) Let $x =$ the acceleration. Since 6 ft. = space passed over in one second; the space passed over in the next second will be 6 ft. + velocity gained in 1 second.

$$\therefore 6 + x = 11$$

$$\therefore x = 5.$$

(12.) Here the velocity acquired in 2 seconds $= (5 \times 5280) \div (30 \times 60) = \frac{44}{3}$

$\therefore$ Acceleration $= \frac{44}{3} \div 2 = 7{\cdot}\dot{3}$ feet.

(13.) By reference to Fig. 101, it will be seen the space described in the 8th second is 15 times (*i.e.*, the 8th odd number) that described in first sec.

$\therefore$ space described in first second $= \frac{570}{15} = 38$

And acceleration $= 38 \times 2 = 76$.

(14.) $s = \frac{1}{2} f t^2 \therefore f = (2 s \div t^2)$

Hence the acceleration $= (784 \div 49) = 16$ ft.

Velocity acquired in 7 secs. $= 16 \times 7 = 112$.

(15.) $s = \frac{1}{2} f t^2 \therefore f = (108 \div 9) = 12$

$\therefore$ the acceleration $= 12$ ft.

Again applying same formula, $\frac{1}{2} \times 12 \times t^2 = 54 + 120 = 174$

$$t = \sqrt{(54 + 120) \div 6} = \sqrt{29} = 5{\cdot}38.$$

And the acceleration $= 5{\cdot}38 - 3 = 2{\cdot}38$.

(16.) Space described in 4th second is 7 times (*i.e.*, the 4th odd number) that during the 1st second, and twice the space in the 1st second is the aceleration

$$\therefore \text{Acceleration} = \frac{70}{7} \times 2 = 20.$$

(17.) Mean velocity $= \frac{43 + 75}{2} = 59$; and time $= \frac{354}{59} = 6$ seconds.

Since velocity gained in 6 seconds $= 75 - 43 = 32$ $\therefore$ acceleration $= 5{\cdot}3$.

(18.) Mean velocity $= \frac{7 + 53}{2} = 30$, and $t = \frac{135}{30} = 4{\cdot}5$ seconds.

Velocity gained per second $= (53 - 7) \div 4{\cdot}5 = 10{\cdot}\dot{2}$.

Now, time from rest $= 53 \div 10{\cdot}\frac{2}{9} = \frac{477}{92}$ seconds.

$$s = \tfrac{1}{2} f t^2 = \tfrac{1}{2} \times 10{\cdot}\dot{2} \times \frac{477}{92} \times \frac{477}{92} = 137\,\frac{73}{184}$$

(19.) Acceleration $= 2 \times 3 \times 4 = 24$.

(20.) Let $x =$ the acceleration; then, in two consecutive seconds, the space passed over in the second is equal to the space passed over in the first second, + the velocity gained in the first.

$$\therefore 36 + x = 44, \text{ and } x = 8.$$

Let $t =$ time from rest, before entering on the first second.

$$\therefore 36 = \tfrac{1}{2} f (t + 1)^2 - \tfrac{1}{2} f t^2 = 4\,(2\,t + 1).$$

$$\therefore t = 4, \text{ and } t + 1 = 5.$$

Page 105.

(1.) $v = g\,t = 32 \times 8 = 256$.

(2.) $v = g\,t$ $\therefore$ $t = 500 \div 32 = 15\frac{5}{8}$ seconds.

(3.) $v = V - g\,t = 80 - 32 \times 3 = -16$ feet (*i.e.*, 16 ft. downwards); and $t = \frac{2\,V}{g} = \frac{160}{32} = 5$ sc.

(4.) $v = V + gt = 160 + (132 \times .5) = 320.$
$v = gt \therefore t = 320 \div 32 = 10$ seconds.

(5.) For A. $v = V + gt = 240 + (32 \times 6) = 432.$
For B. $v = V - gt = 240 - (32 \times 6) = 48.$
The velocities are $\therefore$ as 432 : 48, or 9 : 1.

(6.) $v = V - gt = 96 - (32 \times 4) = -32$ (*i.e.*, 32 ft. downwards).

(7.) Let h be the height; then for the falling body

$$\frac{h}{2} = \tfrac{1}{2} g t^2, \text{ or } t = \sqrt{\frac{h}{g}}$$

For the ascending body $\frac{h}{2} = Vt - \frac{1}{2} g t^2$

Substitute for t, which is the same in both equations,

$$\therefore V = \sqrt{gh}.$$

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(9.) For ascent, $t = \frac{V}{g} = \frac{ag}{g} = a$

For descent, $v = gt' \therefore t' = \frac{bg}{g} = b$

Total time, $a + b$.

(10.) $S = \frac{1}{2} g t^2 = \frac{1}{2} \times 32 \times 49 = 784$ feet.

(11.) $S = \frac{1}{2} \times 32 \times 180^2 = 518{,}400$ feet $=$ 172,800 yards.

(12.) $S = Vt + \frac{1}{2} g t^2 = (50 \times 4\frac{1}{2}) + \frac{1}{2} \times 32 \times (\frac{9}{2})^2$ $= 549$ feet.

(13.) $S = Vt - \frac{1}{2} g t^2 = (150 \times 6\frac{1}{2}) - \frac{1}{2} \times 32 \times (\frac{13}{2})^2$ $= 299$ feet.

(14.) $S = Vt - \frac{1}{2} g t^2 = (96 \times 4{\cdot}75) - \frac{1}{2} \times 32 \times 4{\cdot}75^2 = 95$ feet.

$$\text{Height of ascent} = \frac{V^2}{2g} = \frac{96 \times 96}{2 \times 32} = 144 \text{ feet}$$

$$\text{Time of ascent} = \frac{V}{g} = \frac{96}{32} = 3.$$

Hence, after $4\frac{1}{2}$ seconds, the body will be descending, and will be 95 feet from the ground; hence the whole distance passed over will be $2 \times 144 - 95 = 193$ feet.

(15.) $v = V - gt = 3g - 4g = -g$ (*i.e.*, downwards with velocity g.

$S = Vt - \frac{1}{2} g t^2 = 3g \times 4 - \frac{1}{2} g \times 16 = 4g.$

(16.) See Ex. 4 and 5. Page 103.

(17.) $\frac{V + v}{2}$ = mean velocity. Then $\frac{1}{2}(V + v)t$ = space.

(18.) $\text{Height} = \frac{V^2}{2g} = \frac{36 \times 36}{2 \times 32} = 20\frac{1}{4}$ feet.

(19.) $v^2 = 2gs = 2 \times 32 \times 1600 = 102{,}400$;
and $v = \sqrt{102{,}400} = 320.$

(20.) $S = Vt - \frac{1}{2} g t^2.$
$100 = 100t - 16t^2 \therefore t = 5 \text{ or } 1\frac{1}{4}.$

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(21.) $s = \frac{V^2}{2g} = \frac{80 \times 80}{2 \times 32} = 100$ feet.

(22.) First, $s = \frac{1}{2} \times g \times t^2$, and $t = \sqrt{\frac{81}{16}} = \frac{9}{4}$
Secondly, $s' = \frac{1}{2} \times g \times t^2$, and $t' = \sqrt{\frac{49}{16}} = \frac{7}{4}$
Then the interval $= \frac{9}{4} - \frac{7}{4} = \frac{1}{2}$ second.

(23.) $v^2 = V^2 - 2gs = 60^2 - (2 \times 32 \times 50)$
$v = \sqrt{400} = 20$ feet.

(24.) $v^2 = V^2 + 2gs = 100 + (2 \times 32 \times 75)$
$v = \sqrt{4900} = 70.$

(25.) I. For A, $s = \frac{1}{2} g t^2 = 16 t^2$.

II. $s - 100 = \frac{1}{2} g (t - 1)^2$.

Substituting the first value of s in II.,

$$16 t^2 - 100 = 16 (t - 1)^2$$

$32 t = 116$, and $t = 3\frac{5}{8}$ seconds from A's start.

(26.) $v^2 = V^2 + 2 g s \therefore V = \sqrt{v^2 - 2 g s} = \sqrt{104^2 - (64 \times 69)} = 80$.

(27.) $v^2 = V^2 + 2 f s \therefore f = (v^2 - V^2) \div 2 s = (120^2 - 80^2) \div 200 = 40$.

Page 115.

(1.) $f = g \dfrac{P}{W} = 32 \times \dfrac{2}{12} = 5\frac{1}{3}$ feet.

(2.) $f = g \dfrac{P}{W} = 32 \times \dfrac{1}{50} = \dfrac{16}{25}$ acceleration.

$$s = \tfrac{1}{2} f t^2 = \tfrac{1}{2} \times \frac{16}{25} \times 12^2 = \frac{8 \times 144}{25} = 46 \frac{2}{25}$$

(3.) Velocity gained per second $= \dfrac{120}{6} = 20$

$$f = g \frac{P}{W} \therefore P = \frac{20 \times 12}{32} = 7\tfrac{1}{2} \text{ lbs.}$$

(4.) $f = g \dfrac{P - Q}{P + Q} = 32 \times \dfrac{5 - 3}{5 + 3} = 32 \times \dfrac{2}{8} = 8$.

(5.) $S = \frac{1}{2} f t^2 = \frac{1}{2} \times 8 \times 5^2 = 4 \times 25 = 100$ ft.

(6.) $f = g \dfrac{P - Q}{P + Q} = 32 \times \dfrac{2}{32} = 2$ feet per sec.

$$S = \tfrac{1}{2} f t^2 \therefore 144 = \tfrac{1}{2} \times 2 \times t^2$$

Hence $t = 12$ seconds.

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(7.) $f = g\dfrac{P}{W}$. Now, pressure producing the motion is 2 lbs., and the weight moved is $2 + 50 =$ 52 lbs. $\therefore f = 32 \times \dfrac{2}{52} = \dfrac{16}{13}$

$\therefore$ Velocity acquired $= \dfrac{16}{13} \times 2\frac{1}{2} = = \dfrac{40}{13} = 3\dfrac{1}{13}$.

(8.) $S = \frac{1}{2} f t^2 = \frac{1}{2} \times \dfrac{16}{13} \times 5^2 = \dfrac{8}{13} \times \dfrac{25}{1} = 15\dfrac{5}{13}$ft.

(9.) Let the weight be measured in tons, and the acceleration by miles per hour gained in one hour. Let R = the resistance, and let W = the weight of the second train (including engine). The acceleration in the first case is 60 miles an hour gained in an hour, and in the second case 42 miles an hour gained in an hour. Let P = the force of the engine; then P — R = the pressure producing motion.

$$\therefore 60 = g\frac{P - R}{60}, \text{ and } 42 = g\frac{P - R}{W}$$

$$\therefore g\,(P - R) = 3600 = 42\,W$$

$$\therefore W = 85\tfrac{5}{7} \text{ tons.}$$

PAGE 122.

(1.) Momentum of A $= (10 \times 100) \div g$.
The bodies are brought to rest. $\therefore$ Momentum of

B equals momentum of A. Let x equal velocity in feet of B.

$$\text{Then } (x \times 100) \div g = (10 \times 100) \div g$$

$$x = \frac{10 \times 100}{100} = 10 \text{ feet per second.}$$

(2.) Leaving out g, which cancels, we have

$$6 \times 50 = 300 \text{ momentum of one body.}$$

$$10 \times 20 = 200 \text{ momentum of other body.}$$

Then $\frac{(300 - 200)}{10 + 6} = \frac{100}{16} = 6\frac{1}{4}$ feet per second velocity after impact.

(3.) The momentum of first ball, $5 \times 60 = 300$, must be equal to the total weight of the three balls multiplied by the final velocity,

$$\therefore 300 \div (5 + 7 + 8) = 15 \text{ feet per second.}$$

(4.) Let W_1, W_2, be the weights of the gun and shot; V_1, V_2, their respective velocities.

$$\text{Then } W_1 V_1 = W_2 V_2$$

$$5 \times 2240 \times V_1 = 28 \times 100$$

$$\therefore V_1 = \frac{28 \times 100}{5 \times 2240} = \cdot 25 \text{ feet per second.}$$

(5.) Let x feet per second be velocity of larger body.

Then $x \times 16 = 12 \times 12 \therefore x = 9$ feet per second.

(6.) Let W_1, W_2 bo respective weights; V_1, V_2 the respective velocities. Then $W_1 V_1 = W_2 V_2$. The weights W_1, W_2 are as $2 : 1 \therefore V_1$, V_2 are in inverse proportion, *i.e.*, $1 : 2$.

(7.) Let the velocity of the smaller part after the explosion be less than the original velocity by V_1.

Then the velocity of the larger part will be greater than the original velocity by V_2, such that $W_1 V_1 = W_2 V_2$. Then $30 \times V_1 = 62 \times 30$, and $V_1 = 62$ less than the original V. $\therefore 50 - 62 = -12$, *i.e.*, 12 ft. per second in the opposite direction.

(8.) The momenta gained in the same time by the two must be equal, or $M_1 \cdot f_1 = M_2 \cdot f_2$. Hence the accelerations of the two bodies must be in inverse proportion to their masses.

$\therefore 2 : 5 :: 40 : 100$ acceleration of smaller body.

HYDROSTATICS.

Page 10.

(1 and 2.) See Section 1.

(3.) In the direction of the force only.

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(4.) The pressure will be transmitted undiminished to every point in the sphere.

(5 and 6.) See Solution.

(7.) Pressure on area of $12 = 5$ kilogs.

$$\text{Pressure on area of } 360 = \frac{5 \times 360}{12} = 150 \text{ kilogs.}$$

(8.) Pressure on area of $18^2 = 900$ lbs.

$$\text{Pressure on area of } 3^2 = \frac{900 \times 3^2}{18^2} = 25 \text{ lbs.}$$

(9.) Pressure on area of $4^2 = 400$ grs.

$$\text{Pressure on area of } 55^2 = \frac{400 \times 55^2}{4^2} = 75625 \text{ lbs.}$$

(10.) By application of principle of work (Part I., page 83), we have $x \times 20 = 800 \times 5$.

$x = (800 \times 5) \div 20 = 20$ lbs. $=$ the power.

(11.) Pressure on area of $50^2 = 1000$ kilogs.

$$\text{Pressure on area of } 1^2 = \frac{1000}{2500} = {\cdot}4 \text{ kilogs.}$$

To support 1 gramme a depth of 1 centimetre is required.

To support 4 kilog., (= 400 grammes,) a depth of 400 centimetres, or 4 metres, is required.

(12.) Pressure on a plane of area $36^2 = 2700$.

$$\text{Pressure on a plane of area } 1^2 = \frac{2700}{1296} = 2\tfrac{1}{12} \text{ lbs.}$$

Page 12.

(13.) Pressure on the unit, a square centimetre, = 100 grammes.

Pressure on a surface 150×120 centimetres $= 100 \times 150 \times 120 = 1{,}800{,}000$ grammes.
$= 1800$ kilogs.

(14.) Pressure on area $3^2 = 1$ kilog.

$$\text{Pressure on area } 120^2 = \frac{1 \times 120^2}{3^2} = 1600 \text{ kilogs.}$$

(15.) $6 : 30 :: 100 : 500$ kilogs. Hence, 500 kilogs. is the force acting on the piston; and $1600 \times 500 = 800{,}000$ kilogs. is the force acting on the press.

Page 21.

(1.) Weight of 1 cubic centimetre of water = 1 gramme. In a column 30 metres deep on a square centimetre there will be 3000 cubic centimetres, and the weight of 3000 centimetres = 3000 grammes.

(2.) The pressure on a square inch at a depth of 600 feet is equal to the weight of a column of water the area of the base of which is 1 square inch, and height 600×12 inches; that is, of 7200 cubic inches.

$$\text{Weight of 1 cubic inch} = \frac{1000}{1728} \text{ ozs.}$$

$$\text{Weight of 7200 cubic inches} = \frac{1000 \times 7200}{1728}$$

$= 4166\frac{2}{3}$ ozs.

(3.) Let $x =$ the depth of a column of water on an area of 36 square centimetres whose weight is equal to that of the disc.

$$\therefore x \times 36 = 1080$$

$$\therefore x = 30 \text{ centimetres.}$$

(4.) Area of surface $= 18 \times 15 = 270$.
Pressure at depth of $20 = 270 \times 20 \times 10$ lbs.
$= 54{,}000$ lbs.

(5.) Their edges are as $2 : 1$. Their cubical contents are as $2^3 : 1 = 8 : 1$. $\therefore$ Pressures on their bases are as $8 : 1$.

(6.) Area $= 144$ square inches.
Pressure at depth of $5 = 5 \times 144 \times 8 = 5760$ ozs.
$= 360$ lbs.

(7.) Area of sec. of cylin. $= \left(\frac{6}{2}\right)^2 \times \frac{22}{7} = \frac{9 \times 22}{7}$ square centimetres. Weight of a column of water in the cylinder at the given depth will be $9 \times \frac{22}{7} \times 35$ $= 990$ grammes. Since this is the weight which the pressure of the fluid on the base of the cylinder would support, it is the weight of the plate.

(8.) The pressures on the bases are equal. See Section 22.

(9.) Diagonal of the rectangle $= \sqrt{(11^2 + 60^2)} = 61$ inches.

Area of rectangle $= 11 \times 60 = 660$ square inches.

$$\text{Depth of middle point} = \frac{61}{2} \text{ inches.}$$

Hence pressure is equal to the weight of $660 \times \frac{61}{2}$ or 20,130 cubic inches of fluid $= 20{,}130 \times 1\frac{1}{2} = 30{,}195$ ozs.

(10.) Area of triangle $= \dfrac{50 \times 30}{2} = 750$ square centimetres.

Centre of gravity is at a depth of $\dfrac{30}{3} = 10$ centimetres from the surface.

$\therefore$ Pressure $= 750 \times 10 = 7500$ grammes.

(11.) Area of square in centimetres $= 800 \times 800 = 640{,}000$ square centimetres.

Pressure on 1 sq. cent. of a column 200 cent. high $= 200$ grammes.

Pressure on lid $= 640{,}000 \times 200 = 128{,}000{,}000$ grammes $= 128{,}000$ kilogs.

(12.) $241 - 25 = 216$ ft. height of surface above the tap.

Pressure on 1 sq. ft. at depth of 216 ft. $= 216 \times 1000$

Pressure on 1 sq. in. at ditto $\dfrac{216 \times 1000}{144} = 1500$ ozs.

Page 22.

(13.) The pressure depends simply on the size ot the base and the depth of the liquid. As neither of these will be altered by dipping in the metal, the pressure on the base will remain the same.

(14.) In this case the depth of the water, and therefore, also, the pressure on the base, will be increased. The amount of increase will depend on the shape of the vessel.

Page 28.

(1.) Its specific gravity is the same as that of water,

Therefore weight of 18 cubic centimetres = 18 grams.

(2.) 150 cubic centimetres weigh 120 grammes.

$$1 \text{ cubic centimetre weighs } \frac{120}{150} = \frac{4}{5}$$

Taking specific gravity of water as 1, the specific gravity of the body is $\frac{4}{5}$ or $\cdot 8$.

(3.) 1 centimetre of brass weighs $1 \times 7\cdot 5 = 7\cdot 5$ grammes.

200 centimetres of brass weigh $7\cdot 5 \times 200 = 1500$ grammes.

(4.) Weight of 1 cubic foot of stone $= \frac{24,950}{10}$ $= 2495$ ozs.

Hence the weight of 1 cubic foot of water $= 2494 \div 2\cdot 5 = 998$ ozs.

(5.) Weight of 1 cubic foot of mercury $= 1000 \times 13{\cdot}5 = 13{,}500$ ozs.

Hence $\frac{12 \times 13{,}500}{1728} = 93{\cdot}75$ oz. $=$ wt. of 12 cub. ins.

(6.) Volume of water to weigh 1 gramme $= 1$ cubic centimetre.

Volume of zinc to weigh 1 gramme $= \frac{1}{6{\cdot}85}$ cubic centimetres.

Volume of zinc to weigh $109{\cdot}6 = \frac{1 \times 109{\cdot}6}{6{\cdot}85}$ $= 16$ cubic centimetres.

(7.) Volume of water to weigh 1000 ozs. $= 1$ cub. ft.

Volume of stone to weigh 1000 ozs. $= \frac{1}{2{\cdot}4}$ cubic feet.

Volume of stone to weigh (112×16) ozs. $= \frac{1 \times 112 \times 16}{2{\cdot}4 \times 1000} = {\cdot}746$ cubic feet.

(8.) Specific gravity $= \frac{s_1 v_1 + s_2 v_2 + s_3 v_3}{v_1 + v_2 + v_3}$

$$= \frac{1\frac{1}{2} \times 1{\cdot}2 + 1\frac{1}{3} \times {\cdot}96 + 1\frac{1}{4} \times 1{\cdot}456}{1\frac{1}{2} + 1\frac{1}{3} + 1\frac{1}{4}}$$

$$= \frac{1{\cdot}8 + 1{\cdot}28 + 1{\cdot}82}{4\frac{1}{12}} = 1{\cdot}2.$$

(9.) The weight in water is the true weight minus weight of an equal volume of water, viz., 100 grammes.

$784 - 100 = 684$ grammes $=$ weight in water.

(10.) Weight of 1 cub. cent. of alcohol $= {\cdot}79$ grms.

Weight of 1000 cub. cent. of alcohol $= 790$ grms.

This, too, is the weight of a body which floats with 1000 centimetres of its volume immersed.

(11.) One cubic centimetre weighs the same as $\frac{4}{5}$ of a cubic centimetre of water.

Hence one cubic centimetre weighs $\frac{4}{5}$, or $\cdot 8$ grammes.

(12.) Weight $= 1000 \times 13{\cdot}568$ ozs. $= 848$ lbs.

(13.) Weight of 1000 cubic units of standard substance $= {\cdot}35 \times 1000 = 350$.

Weight of given substance $= 350 \times 3 = 1050$.

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(14.) $$\text{Specific gravity} = \frac{s_1 v_1 + s_2 v_2}{v_1 + v_2}$$
$$= \frac{1{\cdot}3 \times 1 + 3 \times {\cdot}85}{3} = {\cdot}9625.$$

(15.) One cubic centimetre weighs ($\frac{1}{16}$ of 1000) $\div$ 10 $= 6{\cdot}25$ grammes.

Since 1 cubic centimetre of water weighs 1 gramme, the specific gravity $= 6{\cdot}25$.

(16.) Let $x =$ specific gravity of the third.

$$\text{Then } \frac{1{\cdot}55 + 1.75 + x}{3} = 1{\cdot}6$$
$$x = 1{\cdot}5.$$

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(1.) Here $6{\cdot}21$ grammes is loss of weight in water. Hence the specific gravity $= 36{\cdot}84 \div 6{\cdot}21 = 5{\cdot}9$.

(2.) $47{\cdot}48 - 43{\cdot}33 = 4{\cdot}15$ loss of weight in water.

And $47{\cdot}48 \div 4{\cdot}15 = 11{\cdot}4$ specific gravity.

(3.) $$\text{Specific gravity} = \frac{\text{True weight}}{\text{Loss of weight of water}}$$

True weight $=$ specific gravity $\times$ loss of weight in water $= 2{\cdot}56 \times 3{\cdot}78 = 9{\cdot}6768$ grammes.

(4.) Let x = specific gravity of the body.

Loss of weight in water is the true weight divided by the specific gravity.

$$\therefore \frac{5}{x} + \frac{100}{11{\cdot}44} = 9{\cdot}5$$
$$x = 6{\cdot}59.$$

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(5.) Loss of weight in water = 246,000 — 210,000 = 36,000 grammes.

Hence the volume = 36,000 cubic centimetres.

But the volume ought to be $w \div s$, or 246,000 ÷ 8·55 = 28,771·9. There must, therefore, be a cavity or flaw in the substance of the cannon.

(6.) Loss of weight in water = 25 — 20 = 5.

Then specific gravity of body = 25 ÷ 5 = 5.

Loss of weight in the liquid = 25 — 15 = 10.

$$\text{Specific gravity of liquid} = \frac{\text{Loss of weight in liquid}}{\text{Loss of weight in water}}$$
$$= \frac{10}{5} = 2.$$

(7.) Weight of air = 379·895 — 263·525 = 116·37 grammes.

The capacity = weight of whole ÷ weight of a litre
= 116·37 ÷ 1·293 = 90 litres.

(8.) Weight of air in the balloon = 379·895 — 263·525 = 116·37 grammes.

Weight of same volume of gas = 293·687 — 263·525 = 30·162 grammes.

Specific gravity of gas = 30·162 ÷ 116·37 = ·26 nearly.

(9.) Let W = weight of the body. Then $_3$ of its bulk of water weighs W, and the weight of its own bulk of water is 3 W. Specific gravity is therefore W $\div$ 3 W = $\frac{1}{3}$.

(10.) Weight of gold in air = 9·7 grammes.

Weight of gold in water = 104·2 — 95 = 9·2 grs.

Weight of the water which overflows = 9·7 — 9·2 = ·5 grammes.

$\therefore$ Specific gravity = 9·7 $\div$ ·5 = 19·4.

(11.) See Section 37.

Weight of fragment of metal = 14·85 grammes.

Loss of weight in water = 2·03 grammes.

Specific gravity = 14·85 $\div$ 2·03 = 7·31.

(12.) Loss of weight when weighed in pure water is 13 — 6·9 = 6·1 grammes.

Loss of weight when weighed in salt water is 13 — 6·2 = 6·8 grammes.

Now these results give us the weights of equal volumes of pure water and salt water.

$\therefore$ S. G. of salt water = 6·8 $\div$ 6·1 = 1·1147.

(13.) Let a = area of the base; then $10\,a$ = the volume, and $10\,a \times \cdot 25$ the weight.

Let the depth of water equal in weight to 10 inches of cork = x.

$\therefore x\,a = 10\,a \times \cdot 25$, and $x = 2\frac{1}{2}$ inches.

$\therefore$ $2\frac{1}{2}$ inches of cork will be immersed.

(14.) Loss of weight of wood and iron, when weighed in water is (7 + 7·8) — 5·3 = 9·5 lbs.

Loss of weight of iron, when weighed in water, is 7·8 — 6·7 = 1·1 lb.

Loss of weight of wood when weighed in water = 9·5 — 1·1 = 8·4 lbs.

$$\text{Hence specific gravity} = \frac{7}{8{\cdot}4} = \frac{5}{6}.$$

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(15.) Let x = true weight in air.

$$\text{Then specific gravity} = \frac{x}{10}$$

$$\text{But since } W = V \,.\, s \,.\, w$$

$$\therefore x = V \,.\, \frac{x}{10} \cdot 1000$$

$$\therefore V = {\cdot}01.$$

(16.) Let W = weight of body; then $\frac{1}{3}$ of its bulk of liquid = W, and the weight of its own bulk of liquid is therefore 3 W, its specific gravity then referred to liquid is $\frac{1}{3}$, and referred to water = $\frac{1}{3} \times 2{\cdot}5 = {\cdot}83$.

(17.) Let W = weight of body.

Then $\frac{2}{3}$ of its bulk of the liquid = W

$$\text{And its own bulk of liquid} = \frac{3\,W}{2}$$

$$\therefore \text{Specific gravity} = \frac{1}{3} \times 2{\cdot}5 \times \frac{3}{2} = 1{\cdot}25.$$

(18.) The water will exert a pressure on the metal equal to the weight of 10 cubic centimetres of water; that is, to 10 grammes. Hence, also, the metal will exert a pressure on the water of 10 grams.

(19.) Since the specific gravity of the metal is 8·5

8·5 lbs. of metal lose 1 lb. in water

$\therefore$ 8·5 lbs. of metal weigh 7·5 in water

$$20 \text{ lbs. of metal weigh } \frac{7{\cdot}5 \times 20}{8{\cdot}5} = 17\tfrac{11}{17} \text{ lbs.}$$

(20.) Let w = the weight of the rod, and v its volume. Then $\frac{v}{10}$ = volume of rod. The weight of stone having a volume $\frac{v}{10}$ = weight of water having a volume $\frac{v}{4} = \frac{w}{3}$.

Hence weight of a volume v of stone $= \frac{10}{3}w$.

The specific gravity of the stone as referred to water is therefore $\frac{10}{3} \div \frac{4}{3} = 2{\cdot}5$.

(21.) None. The upward pressure is equal to the downward pressure; and both, in this case, are equal to the weight of the body.

(22.) Weight of body is equal to the weight of water displaced, viz., 5·76 cubic inches.

Then weight in lbs. $= \frac{5{\cdot}76 \times 1000}{1728} \div 16 = \frac{5}{24}$ lbs.

Total weight $= 10\frac{5}{24}$ lbs.

(23 and 24.) Let v_1 be the volume of a, and v_2 that of b, and let $v_1 > v_2$.

Let w_1 be the weight of a, and w_2 that of b.

Now, the weight of a body = its weight in water + the weight of the water it displaces.

Hence, since both a and b weigh the same in water, but a displaces more than b, $w_1 > w_2$.

Let w_3 be the weight of both in water, and x and y their weights in mercury of specific gravity g.

The specific gravity of the two bodies are therefore

$$\frac{w_1}{w_1 - w_3} \text{ and } \frac{w_2}{w_2 - w_3}$$

$$\text{They are also } \frac{w_1 g}{w_1 - x} \text{ and } \frac{w_2 g}{w_2 - y}$$

$$\therefore g(w_1 - w_3) = w_1 - x, \text{ and } g(w_2 - w_3) = w_2 - y$$

$$\therefore x = g\,w_3 - (g - 1)\,w_1, \text{ and } y = g\,w_3 - (g - 1)\,w_2$$

$$\text{But } w_1 > w_2$$

$$\therefore x \quad y$$

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(1.) Water is 13·596 times lighter than the mercury used in the barometer; therefore, a column of $76 \times 13{\cdot}596 = 1033{\cdot}296$ centimetres is supported.

(2.) Height of column if liquid were water $= 76 \times 13{\cdot}596$ centimetres.

Height of column of liquid of specific gravity 2·5 $= (76 \times 13{\cdot}596) \div 2{\cdot}5 = 413{\cdot}3$ centimetres.

(3.) Height of column of alcohol $= 33 \div {\cdot}825 = 40$ feet.

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(4.) Refer to Fig. 27, page 56. The pressure at B = atmospheric pressure (P) — pressure due to column of height D E. If a small hole be made at B, the fluid at B will be exposed to the pressure of the atmosphere P, and the resultant pressure at B will be P — P — pressure due to height D E, and will therefore cause the water to descend in B D. In a similar manner the water will descend in B C.

(5.) The vertical difference of level between the surfaces of the fluid in the cistern and tube will remain the same. Hence, the fluid will occupy a greater length of tube.

(6.) Refer to Fig. 27. The greatest height of D E at any station is the height of a barometer of the fluid at that station. As the pressure of the atmosphere diminishes as we ascend a mountain, the limit of the length D E diminishes.

(7.) Weight of cubic inch of water $= \frac{1000}{1728}$ ozs.

Column of water supported $= 30 \times 13\cdot5$ inches.

Pressure per square inch $= \frac{1000}{1728} \times 30 \times 13\cdot5$ oz.

The air is half exhausted, therefore the pressure is diminished by one-half.

Pressure $= \frac{1000}{1728} \times 30 \times 13\cdot5 \div 2 = 117\cdot18$ oz.

(8.) Pressure on square inch by column 1 inch high $= 7\cdot8$ ozs.

Pressure on square inch by column $29\cdot5$ inches high $= 7\cdot8 \times 29\cdot5 = 230\cdot1$ ozs.

(9.) Column of water supported $= 29\cdot5 \times 13\cdot5 = 398\cdot25$ inches.

(10.) Let $x =$ height of mercury;

$3\,x =$ height of water;

$13\cdot596\,x =$ height of water equal to x centimetres of mercury.

A water barometer would stand at 1020 centimetres.

$\therefore\ 3x + 13\cdot596\,x = 1020$ centimetres.

$x = 61\cdot5$ centimetres.

(11.) See Question 6. The limit of the height D E will be diminished.

(12.) The pressure of the atmosphere is transmitted by the fluid to the sides and base. Hence, when the barometer falls, this pressure will diminish,

and when the barometer rises, the pressure will increase.

(13.) Height of a water barometer $= 760 \times 13 \cdot 5$ millimetres.

Hence, height of a barometer of the given liquid $= (760 \times 13 \cdot 5) \div 3 \cdot 5 = 2931 \cdot 4$ millimetres.

(14.) See Solution.

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(15.) Total pressure at depth of 45 inches is the weight of $(30 \times 13 \cdot 5) + 45 = 450$ inches of water $= \frac{450}{13 \cdot 5}$, or $33\frac{1}{3}$ inches of mercury.

(16.) Taking contents of the barrel as 1 and that of the receiver as 5 $\therefore$ The density after the first stroke will be $\frac{5}{6}$ of the original density; after the second it will be $\frac{5}{6}$ of $\frac{5}{6} = \frac{25}{36}$ of original density; and after the third stroke $\frac{5}{6}$ of $\frac{25}{36} = \frac{125}{216}$; *i.e.*, the density has been diminished by nearly a half.

(17.) After first stroke, density $= \frac{5}{6}$; after second, density $= \frac{5}{6}$ of $\frac{5}{6} = \left(\frac{5}{6}\right)^2$; after third, $\frac{5}{6}$ of $\left(\frac{5}{6}\right)^2 = \left(\frac{5}{6}\right)^3$; and so on.

$\therefore$ Taking original volume as 1, the air removed is $1 - \left(\frac{5}{6}\right)^6$.

(18.) Since the pressure of the air on the inside of the top is the pressure at the surface of the water in the bell, and the pressure on the outside of the top is the pressure due to the depth of the top, the former pressure exceeds the latter; and, if a hole be made in the top, the air will be forced through it.

(19.) 12 metres = 1200 centimetres. Hence, the pressure on a square centimetre is that of a column of water of the height of $(76 \times 13{\cdot}596) + 1200 = 2233{\cdot}296$ grammes = 2·233,296 kilogrammes.

(20.) See Solution to Question 14.

As the air surrounding the body is made denser, the weight of the displaced air increases, and therefore the weight of displaced water must decrease; or, in other words, less of the body will be immersed.

THE END.

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